Homework #1 Math 518
Due in class Wednesday, August 31, 2011

Exercise 1.1. a. Let M be a manifold, $V \subset M$ an open subset. Prove that if $\{\varphi_\alpha : U_\alpha \to \varphi_\alpha(U_\alpha) \subset \mathbb{R}^n\}$ is an atlas on M then $\{\varphi_\alpha|_{V \cap U_\alpha}\}$ is an atlas on V. Conclude that V is a manifold.

b. Show that

$$GL(n, \mathbb{R}) := \{A \text{ an } n \times n \text{ matrix } \mid \det A \neq 0\}$$

is an n^2 dimensional manifold.

Exercise 1.2. Prove that for each matrix $A \in GL(n, \mathbb{R})$ the map

$$L_A : GL(n, \mathbb{R}) \to GL(n, \mathbb{R}), \quad L_A(B) := AB$$

is a diffeomorphism.

Exercise 1.3. Check that a notion of a smooth map between manifolds (definition 2.17 in notes) does not depend on the choice of charts or atlases and therefore is well-defined.