Homework #10 Math 518
Due in class Wednesday, November 2, 2011

1. Let G be a Lie group. For a vector $X \in \mathfrak{g} = T_1G$ denote by γ_X the integral curve through 1 of the corresponding left-invariant vector field $\tilde{X}$. Show that the map

$$\exp : \mathfrak{g} \to G, \quad X \mapsto \gamma_X(1)$$

is smooth. This map is called the exponential map (does it have anything to do with the exponentiation of matrices?). Hint: consider the vector field V on $G \times \mathfrak{g}$ defined by

$$V(a, X) = ((dL_a)_1 X, 0).$$

Check that the flow Φ of V is given by

$$\Phi(a, X, t) = (a\gamma_X(t), X).$$

2. Calculate the exterior derivatives of the following forms in $\mathbb{R}^3$:

i. $z^2\, dx \wedge dy + (z^2 + 2y) dx \wedge dz$;

ii. $13x\, dx + y^2\, dy + xyz\, dz$;

iii. $x^2 y^3 z^6 dx \wedge dy \wedge dz$.

3 Suppose $\pi : E \to B$ is a vector bundle and $f : Q \to B$ a map of manifolds.

a. Prove that $f^*E := \{(q, e) \in Q \times E \mid f(q) = \pi(e)$ is a submanifold of $Q \times E$. Hint: homework #4, problem 3.

b. Prove that the map $\pi' : f^*E \to Q$, $\pi'(q, e) = q$, makes f^*E into a vector bundle over Q of the same rank as E. Hint: where would the trivialization maps of f^*E come from?

4 Let $f : E \to F$ be a map of vector bundles over a manifold B which is an isomorphism on each fiber. Show that f has a *smooth* inverse.