Homework #14 Math 518
Due in class Wednesday, December 7, 2011

1. Let D be an m-dimensional oriented domain with boundary ∂D (equivalently, a manifold with boundary). Suppose α, β are two differential forms on D such that

1. $\beta|_{\partial D} = 0$ and
2. $|\alpha| + |\beta| = m - 1\,(= \dim D - 1)$.

Prove that

$$\int_D d\alpha \wedge \beta = -(-1)^{|\alpha|} \int_D \alpha \wedge d\beta.$$

2. Let D be an m-dimensional oriented domain with boundary ∂D (equivalently, a manifold with boundary), $F : \partial D \to N$ a map of manifolds and $\omega \in \Omega^{m-1}(N)$ a closed form (i.e., $d\omega = 0$). Prove that if F can be extended to a smooth map $\tilde{F} : D \to N$ then

$$\int_{\partial D} F^*\omega = 0.$$

3. What is

$$\int_{S^2} \omega,$$

where $S^2 = \{(x, y, z)\ in\mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$ is the standard unit sphere in $\mathbb{R}^3$ oriented as the boundary of the unit ball (i.e., outward normal) and

$$\omega = (3x^2 \cos(y) + e^{xy})\, dx \wedge dy + 17x^3\, dx \wedge dz + (x + yz^3 + \sin(z))\, dy \wedge dz \quad ?$$