Homework #3 Math 518
Due in class Wednesday, September 14, 2011

1. Let M be a manifold, $p \in M$ a point and $v \in T_pM$ a tangent vector at the point p. Show that there is a curve $\gamma : I \to M$ (where I is an open interval containing 0) with $\gamma(0) = p$ and $\dot\gamma(0) = v$. Hint: Why is it true for $M = \mathbb{R}^n$?

2(a). Prove that if $F : M \to N$ is a diffeomorphism, then for any $p \in M$ the linear map $dF_p : T_pM \to T_{F(p)}N$ is an isomorphism.

2(b). Prove that if

$$d(\det)_I : T_I GL(n, \mathbb{R}) \to T_1\mathbb{R} \simeq \mathbb{R}$$

is onto then so is

$$d(\det)_A : T_A GL(n, \mathbb{R}) \to T_1\mathbb{R}$$

for any matrix A with $\det A = 1$. Hints: part (a) and an exercise from Homework 1 may be useful; drawing a commuting triangle of maps may also be helpful.

2(c) Prove that the differential $d(\det)_I$ is the trace. Use this to prove that $d(\det)_I : T_I GL(n, \mathbb{R}) \to T_1\mathbb{R}$ is onto. Now prove that 1 is a regular value of $\det : GL(n, \mathbb{R}) \to \mathbb{R}$.