**Homework #5v2 Math 518** (small correction in #4 on 9/22)
**Due in class** Wednesday, September 28, 2011

1. Prove that for any manifolds $M$ and $N$ the tangent bundle $T(M \times N)$ is diffeomorphic to $TM \times TN$.

2. Let $A$ be a vector space over $\mathbb{R}$ with an associative multiplication (for example, $A$ could be the vector space of $n \times n$ matrices with matrix multiplication). That is, let $A$ be an *algebra* over $\mathbb{R}$. Define a bracket on $A$ by

$$[a, b] := ab - ba$$

for all $a, b \in A$. Here $ab$ and $ba$ denote the products in $A$. Prove that $A$ with this bracket is a Lie algebra.

3. Check that $\Phi(t, x) = \frac{x}{1-xt}$ is the flow of the vector field $x^2 \frac{d}{dx}$ on $\mathbb{R}$.

4. Prove that if $F : M \to N$ is a diffeomorphism, then so is the map $dF : TM \to TN$ defined by

$$dF(p, v) := (F(p), dF_p(v)) \text{ for all } p \in M, v \in T_pM.$$