Homework #8 Math 518
Due in class Wednesday, October 19, 2011

1. Let V, W be two finite dimensional vector spaces over the reals.
a. Show that there is a natural isomorphism $\phi : V^* \otimes W^* \xrightarrow{\simeq} \text{Mult}(V, W; \mathbb{R})$ with

$$\phi(v^* \otimes w^*)(v, w) = v^*(v)w^*(w)$$

for all $v^* \in V^*$,$w^* \in W^*$, $v \in V$,$w \in W$.
a. Show that there is a natural isomorphism $\psi : V^* \otimes W^* \to (V \otimes W)^*$ with

$$\psi(v^* \otimes w^*)\,(v \otimes w) = v^*(v)w^*(w)$$

for all $v^* \in V^*$,$w^* \in W^*$, $v \in V$,$w \in W$.

2. Let V_1, V_2, V_3 and U be finite dimensional vector spaces over the reals. Show that for any multilinear map

$$\mu : V_1 \times V_2 \times V_3 \to U$$

there exists a unique linear map $\bar{\mu} : V_1 \otimes (V_2 \otimes V_3) \to U$ so that

$$\mu(v_1, v_2, v_3) = \bar{\mu}(v_1 \otimes (v_2 \otimes v_3))$$

for all $(v_1, v_2, v_3) \in V_1 \times V_2 \times V_3$.

3. Show that the map $\mathbb{R} \times V \to V$, $(a, v) \mapsto av$ gives rise to an isomorphism $\mathbb{R} \otimes V \xrightarrow{\simeq} V$ which sends $a \otimes v$ to av for all $a \in \mathbb{R}$ and $v \in V$.

4. Suppose that V is an n-dimensional vector space. Given a linear map $A : V \to V$, we get a map $\Lambda^n(A) : \Lambda^n(V) \to \Lambda^n(W)$, and since $\dim \Lambda^n(V) = 1$, the map $\Lambda^n(A)$ is multiplication by a scalar. Show that this scalar is $\det A$.