Homework #9 Math 518
Due in class Wednesday, October 26, 2011

1. Let G be a Lie group, X a left invariant vector field of on G and $\gamma : \mathbb{R} \to G$ an integral curve of X with $\gamma(0) = 1$ (here and elsewhere 1 denotes the identity element of the group G). Prove that for all $s, t \in \mathbb{R}$

$$\gamma(s + t) = \gamma(s) \cdot \gamma(t),$$

where on the right $\cdot$ denotes the multiplication in G. In other words prove that integral curves through 1 of left invariant vector fields are Lie group homomorphisms. They are called 1-parameter subgroups.

2. Recall that for a Lie group the tangent space at the identity has the structure of a Lie algebra. Prove: if $f : G \to H$ is a map of Lie groups (i.e., f is smooth and preserves multiplication) then

$$df_1 : T_1G \to T_1H$$

is a map of Lie algebras.

3. Let $S = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_4 = x_1 x_2 x_3^2,\ 0 < x_1, x_2, x_3 < 1\}$. It's a submanifold of $\mathbb{R}^4$ and the map $\varphi(x_1, x_2, x_3, x_4) = (x_1, x_2, x_3)$ is a coordinate chart on S giving it an orientation (don't prove these two facts). Compute

$$\int_S x_3 dx_1 \wedge dx_2 \wedge dx_4,$$

where S is oriented as above.

4. Compute the pull-back of the form

$$\omega = x dy \wedge dz \wedge dw - y dx \wedge dz \wedge dw + z dx \wedge dy \wedge dw - w dx \wedge dy \wedge dz \in \Omega^*(\mathbb{R}^4)$$

by the map $F : \mathbb{R}^3 \to \mathbb{R}^4$, $F(x, y, z) = (x, y, z, x^2 + y^2 + z^2)$.